# **Constitutive Model and Finite Element Procedure for the Analysis of the Inelastic Behavior of Steel Columns in Fire**

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(Received 30 November 2009; accepted 7 December 2009)

In this paper, a mechanical model is presented of structural steel at high temperature, and a numerical simulation is conducted of a fire-resistance test of large steel columns. Based on a detailed investigation of the material data obtained from various creep experiments, a computer-oriented mechanical behavior model for structural steel was developed. The results of the calculations clearly show that numerical analysis can be used to provide accurate predictions of the deformation characteristics of columns at high temperatures when an appropriate mechanical model and creep strain data of steel materials at high temperatures are used.

Key words: High temperature, Mechanical model, Creep deformation, Steel column, Fire-resistance test

# 1. INTRODUCTION

The success of analyzing the behavior of steel structures in a fire condition depends on the accuracy with which the mechanical properties, especially the creep of steel, are known in the range of 500 to 600 Centigrade. In this study, the inelastic behaviors of protected steel columns exposed to a standard furnace fire test are investigated numerically by the method of finite element elasto-plastic-creep analysis, including the constitutive model proposed by Furumura et al. [1] and the geometrical nonlinearities of steel members.

# 2. MODELING STEEL BEHAVIOR

It is generally agreed that the deformation process of steel at transient high temperatures can be described by three strain components defined by the constitutive equation

$$\varepsilon = \varepsilon_{\sigma}(\sigma, T) + \varepsilon_{th}(T) + \varepsilon_{cr}(\sigma, T, t)$$
(1)

where

 $\varepsilon_{\sigma}$  = instantaneous stress-related strain based on stress-strain relations obtained under constant stabilized temperature.

 $\varepsilon_{cr}$  = creep strain or time-dependent strain.

 $\varepsilon_{th}$  = thermal strain

A computer-oriented mechanical behavior model for structural steel, based on Eq.(1), was developed by Furumura et al.[2]. The strains are found separately in different steadystate tests. It is known that a behavior model based on steadystate data satisfactorily predicts behavior in transient test under any given fire process, load and strain history.

#### 2.1 Instantaneous Stress-related Strain

The analytical model of instantaneous  $\sigma$ - $\epsilon$  curves at con-





Fig.1 Stress-strain relationship (SN490B)



Fig.2 Equations of stress-strain relationship

stant temperatures is shown in Fig. 1. The  $\sigma$ - $\epsilon$  relationship can be measured by the tensile test under strain rate control. In Reference [4], static tensile tests were conducted, and the stress-strain curves from ambient temperature to 700 C were obtained. To represent these stress-strain curves mathematically, Eqs. (2) and (3) in Fig. 2 were adopted. The coefficients on Eqs. (2) and (3) in Fig. 2 were determined by applying the least squares method to the  $\sigma$ - $\epsilon$  curves directly measured by



Fig.3 Experimental creep strain (Reference [3])



Fig.4 Stress-strain curves accompanied with creep strain.

the tensile test.

### 2.2 Creep Strain

#### 2.2.1 Steady-state creep strain

Creep formulations of steel at the steady-state were developed by Furumura et al. [1-3], and the following formula for structural steel SM50 was presented:

•• $e_{\sigma}(\sigma, T, t) = 10^{a/T+b} \cdot \sigma^{c/T+d} \cdot t^{eT+f}$  (for SM50A), • • •(4) where  $\varepsilon_{cr}$  is the creep strain (%), t is the time (minutes),  $\sigma$  is the stress (N/mm<sup>2</sup>), and T is the absolute temperature (K). The material constants of SM50 were as follows:  $a = -8.48 \times 10^3$ , b = 4.50,  $c = 3.06 \times 10^3$ ,  $d = 2.28 \times 10^{-1}$ ,  $e = 2.00 \times 10^{-3}$ , and f = -1.10.

This creep-strain equation at constant high temperatures was determined by conducting creep tests and approximated using the least squares method to determine the coefficients of Eq.(4). In Fig. 3, the dashed lines show the experimental values of creep tests at high temperatures, and the solid lines show the values calculated using Eq.(3). The relationship as a whole showed experimental values taking over the calculated values in the middle of the course.

#### 2.2.2 Modified Strain-hardening Creep Law

Based on the detailed investigations of the data obtained from the various creep experiments, the effect of variable stress and temperature for creep strain was solved by Furumura et



Fig.5 Stress-strain curves accompanied with creep strain



Fig.6 Modified strain-hardening creep law

#### al. [1] using the modified strain-hardening law.

The stress-strain curves obtained, together with the creep curves, are shown in Fig. 4. Although plastic strain is defined as time-independent, it seems reasonable to take plastic strain as well as creep strain into consideration when the strain hardening law is applied. The strain-hardening law in which plastic strain was taken into account as the same kind of strain as creep was called the modified strain-hardening law.

This law is clearly shown in Figs. 5 and 6. If, for instance, the stress is raised from a constant stress  $\sigma_1$  to a higher value  $\sigma_2$  after a certain time period  $t_1$ , the creep strain caused during the time period  $t_1$  is O1B, and, at the time  $t_1$ , the values of the stress and strain change from those of B to those of C in Fig.4. After that, the creep strain follows the curve CJ(Fig.6), and, after a lapse of time,  $t_1+t_2$ , the total creep strain becomes O1B+CJ.

Using the modified strain-hardening law, it seems that the nonlinear elasto-plastic creep deformation of steel structures under continuously varying stresses and temperatures can be evaluated with reasonable accuracy.

#### 2.3 Thermal Strain



Fig.7 Loaded fire resistance test (Reference [4])

The free expansion of steel due to temperature change  $\varepsilon_{\tau}$  is obtained as follows:

 $\varepsilon_T = 5.04 \text{x} 10^{-9} \text{x} T^2 + 1.13 \text{x} 10^{-5} \text{x} \text{T}$  (5)

where T is in degrees Centigrade.

# 3. COMPUTER SIMULATION OF STEEL COLUMNS DURING FIRE TESTS

## 3.1 Loaded Fire Resistance Test of Steel Column

An overview of the loaded fire resistance tests conducted in Reference [4] is shown in Fig. 7. Two press-formed square steel columns, B-CS06 and B-CS10, were used as test specimens. Test column specimens were fabricated using SN490B grade steel, and the measured value of the steel yield strength at ambient temperature was 363MPa. The sectional shape of the columns was  $\Box$  -600x600x28.

The existing load ratios to the sustained allowable load were 0.6 for B-CS06 and 1.0 for B-CS10. The steel column B-CS06 was heated by the ISO 834 standard fire temperature curve, and B-CS10 was heated by the hydrocarbon fire curve.

A full-scale fire test on a steel column exposed to fire that was tested by Kohno et al.[4] was simulated by the finiteelement method based on the beam theory.

Since no experimental studies have been conducted on the creep strain at high temperatures for SN490B, the creepstrain equation of SM50A, which should have similar creep properties, was used to estimate the creep strain for SN490.

#### 3.2 Analytical Results and Discussion

The upper figures in Fig. 8 and 9 show the steel



Fig.8 Predicted and measured deformation (B-CS06)

temperatures measured at points on Level E (experimental values), the middle figures show column the elongations (experimental and calculated values), and the bottom figures show the deflection *W* in the middle of the columns (calculated values). To examine the effects of the accuracy of the creep-strain equation at high temperatures, three cases were calculated using values 40% larger and smaller than the value determined by Equation (5). In the middle figures, open circles denote the experimental values, solid lines denote values calculated by considering creep at high temperatures, and dashed lines denote calculations made by disregarding the creep strain.

As shown in the upper figures, the temperature distribution was almost uniform throughout the cross section when the columns were heated. The steel temperatures of B-CS06 and B-CS10 rose almost linearly with a temperature rise



Fig.9 Predicted and measured deformation (B-CS10)

### of 2.5 to 3.3 K per minute.

The calculated values for B-CS06 with consideration of the creep strain at high temperatures (solid line) reproduced the experimental values very accurately. The 40% fluctuation of the creep strain at high temperatures corresponded to a collapse time of about 10 minutes. The values calculated by disregarding the creep strain at high temperatures (dashed lines) continued rising until immediately before the collapse and differed from the actual contraction observed in the experiment.

The results show that, at high temperatures, creep strain needs to be considered to correctly reproduce the actual behaviors during collapse (contraction of displacement U) and that disregarding creep strain results in predicting collapse time on the dangerous side.

The calculated values for B-CS10 (Fig. 9) with

consideration of the creep strain at high temperatures (solid line) reproduced the experimental results except that the calculated contraction behavior occurred slightly earlier than the actual contraction (open circles). Since the 40% reduction in the creep strain at high temperatures matched the experimental values, Eq. (3) for calculating the creep strain at high temperatures may tend to overestimate the creep strain of SN steel under large stresses.

The values calculated by disregarding the creep strain at high temperatures (dashed lines) continued rising until immediately before the collapse and differed from the actual contraction observed in the experiment, suggesting that creep strain at high temperatures cannot be disregarded if the experimental collapse behavior is to be reproduced. However, the calculated collapse time (dashed line) nearly agreed with the actual collapse time.

The bottom figures in Figs. 8 and 9 show the calculated deflection W in the middle of the columns and show that deflection W started to increase when axial displacement U started to contract.

# 4. CONCLUSIONS

A non-linear simulation analysis of full-scale square steel tube columns was conducted using a constitutive model of steel at elevated temperatures developed by Furumura et al. The onedimensional finite-element calculation method used in this study appears to be valid, and the equation for calculating creep strain at high temperature for SM50A steel also appears to be feasible for predicting the behaviors of SN490 steel at high temperatures.

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